The Role of Content In Value-Added Research

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Curriculum Content: A serious Challenge For Value-Added Studies

- The Measurement Invariance Requirement
- Bias in the estimation of Teacher/School/District effects

Curriculum Content: A serious Challenge For Value-Added Studies

In both cases, the challenge is to the Strong Assumption underlying most value added research:

Content Homogeneity

both across grades and across classrooms, schools and districts.

Curriculum Content: A serious Challenge For Value-Added Studies

Vertical scaling – critical component for the validity of value added studies depends on the Measurement Invariance Assumption.

To measure change *"implies … that … identical scores on the two scales are to be interpreted as having identical meaning …"* (Lord, 1963)

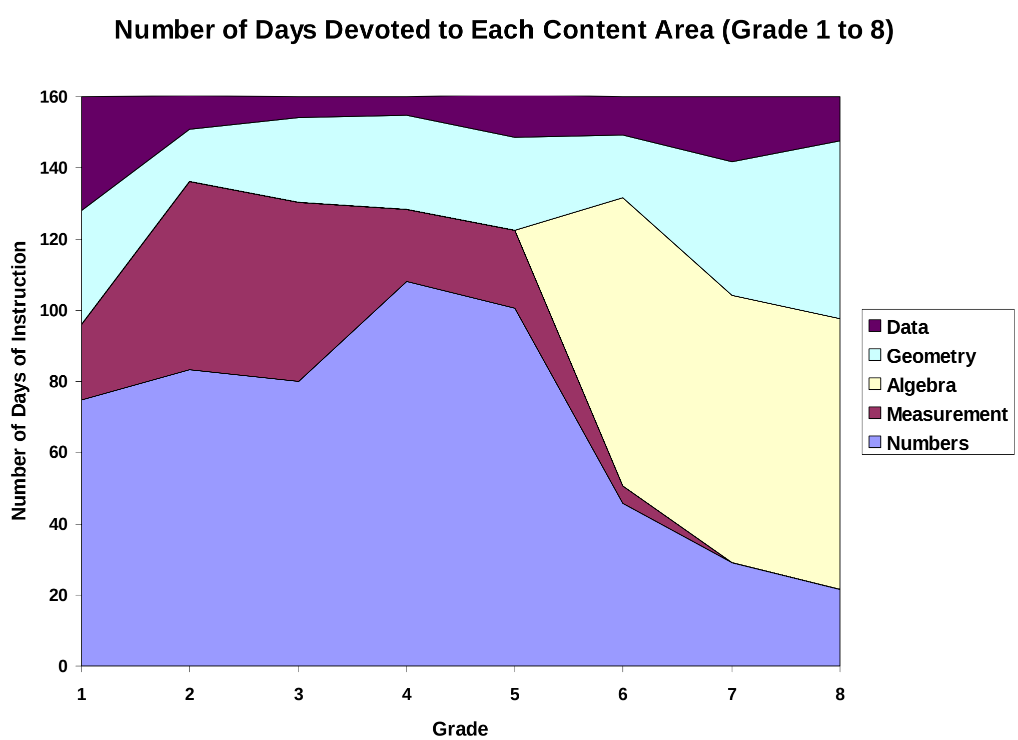
Put simply, the tests must measure the same thing at each time point for the measurement of change or growth.

Curriculum Content: A serious Challenge For Value-Added Studies

The Conceptual or Discipline based Challenge to the Measurement Invariance Assumption is that

Math is *NOT* Math

Michigan Mathematics Standards



Michigan Mathematics Standards

Grade

1 → **9**

Whole Number Arithmetic

Real Number System And Its Axioms

Algebra And Fractions

Michigan Mathematics Standards

Grades 1 to 5 - Add, subtract, multiply, divide whole numbers and fractions

Grades 6 to 8 - Add, subtract, multiply, divide rational numbers

- Two tests defined on these domains might appear to be very similar.
- But from a mathematics point of view, we know them to be different.
- This would violate the Measurement Invariance requirement and seriously challenge the validity of vertical scaling.

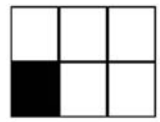
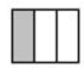
Michigan Math Standards – Fractions Grades 2-4

Grade	Standard	Remarks
2	**PLACE 0 AND HALVES (1/2, 1 1/2, 2 1/2, ETC.) ON THE NUMBER LINE.**	• Have students use the ruler as a model of the number line.
3	**KNOW THAT ONCE A WHOLE HAS BEEN FIXED OR DEFINED, FRACTIONS REFER TO EQUAL PARTS OF THE WHOLE.**	• The representation of the whole can be the length of an interval on the number line, the area of a rectangular strip or the area of a square.
		• Until a whole is made explicit in context, a fraction has no meaning.
3	**RECOGNIZE, NAME AND USE EQUIVALENT FRACTIONS WITH DENOMINATORS 2, 4 AND 8.**	• Include the terms 'numerator' and 'denominator.'
		• In order to understand equivalent fractions, such as 1/2 = 2/4, remind students that the whole is the unit. Without fixing the whole, equivalent fractions have no meaning.
		• Use strips.
3	**ADD AND SUBTRACT FRACTIONS WITH THE SAME DENOMINATOR.**	• Emphasize the similarity of the addition and subtraction of fractions with the same operations on whole numbers.
		• For example, 12 = 4x3 = 2x6 = 2x2x3.
4	**LOCATE AND COMPARE FRACTIONS WITH DENOMINATORS OF 12 OR LESS ON THE NUMBER LINE.**	• Note this involves improper fractions.
4	**ADD AND SUBTRACT FRACTIONS LESS THAN 1 AND WITH DENOMINATORS UP TO 12 OR EQUAL TO 100.**	• Illustrate using fractional parts of rectangles.
		• **Exclude** sums involving more than 2 different denominators.

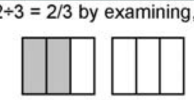
Michigan Math Standards – Fractions Grades 4-5

Grade	Standard	Remarks
		• Exclude sums where one denominator is not a multiple of the other.
		• Use the number line extensively.
		• Take verbal statements of a problem and have students write mathematically.
		• Continue to use letters to represent unknowns.
5	**GIVEN TWO FRACTIONS, EXPRESS THEM AS EQUIVALENT FRACTIONS WITH A COMMON DENOMINATOR BUT NOT NECESSARILY A LEAST COMMON DENOMINATOR (EMPHASIS ON DENOMINATORS EQUAL TO OR LESS THAN 12 OR EQUAL TO 100).**	• The easiest common denominator of a/b, c/d is bd.
		• Include reducing fractions to lowest terms.
5	**ADD AND SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS; DENOMINATORS OF GIVEN FRACTIONS ARE EQUAL TO 1,2,..,11,12, OR 100.**	• Take verbal statements of a problem and have students write mathematically.
		• Include listing of equivalent fractions to identify fractions with common denominator.
		• Denominators of given fractions should not exceed 12.
		• Do not focus on the formula per se - students should not be required to memorize the formula in its abstract form, but to understand its use
		• Help students understand why this algorithm works.

Michigan Math Standards – Fractions Grades 5

Grade	Standard	Remarks
5	KNOW THE MEANING OF THE PRODUCT OF TWO UNIT FRACTIONS IN TERMS OF AN AREA MODEL AS WELL AS THE PRODUCT OF A FRACTION BY A WHOLE NUMBER.	• Emphasize fractions with small denominators for the purpose of drawing pictures. For example, $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ 
		• Do not use pie models here.
		• Help students understand that multiplication of a number by a fraction can result in a smaller number.
5	UNDERSTAND A FRACTION AS A STATEMENT OF DIVISION.	• Show that 1 ÷ 3 = 1/3 by examining:  where the area of the square is defined as the whole for interpreting the fraction. The picture represents 1 part of 3, 1/3 and 1÷3.

Michigan Math Standards – Fractions Grades 5

Grade	Standard	Remarks
		• Show that 2÷3 = 2/3 by examining,  where the area of one square is defined as the whole for interpreting the fraction 2/3. Interpret 2÷3 as the size of a part that results when 2 units are divided into 3 equal parts.
		• For example, 2/3 is the division of 2 by 3 and can be described by the property that 3 times 2/3 is 2.
		• For example, you have 3 cookies to divide among 4 people. Each person gets 3/4 of a cookie. Therefore, 3÷4 = 3/4.
		• For example, recognize that 2/3 implies 2 out of 3 parts, but also 2÷3 in the general sense of partitive division.
		• Include conversion between fractions and decimals.

Michigan Math Standards – Fractions Grades 6

Grade	Standard	Remarks
6	**DIVIDE ANY TWO FRACTIONS INCLUDING MIXED NUMBERS.**	• Emphasize that, as in the case of whole numbers, division of fractions is just a rewrite of the corresponding statement about multiplication of fractions. Thus, if A,B,C are fractions, then *A ÷ B = C* is another way of writing *A = C x B*. Thus, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \Rightarrow \frac{ad}{bc} \times \frac{c}{d} = \frac{a}{b}$ And we see that ad/bc is the quantity which, when multiplied by c/d gives a/b.
		• There will be a tendency to simply memorize the formula "invert and multiply." It is very important that this formula be explained and justified, not just memorized.
		• Take verbal statements of a problem and have students write mathematically.

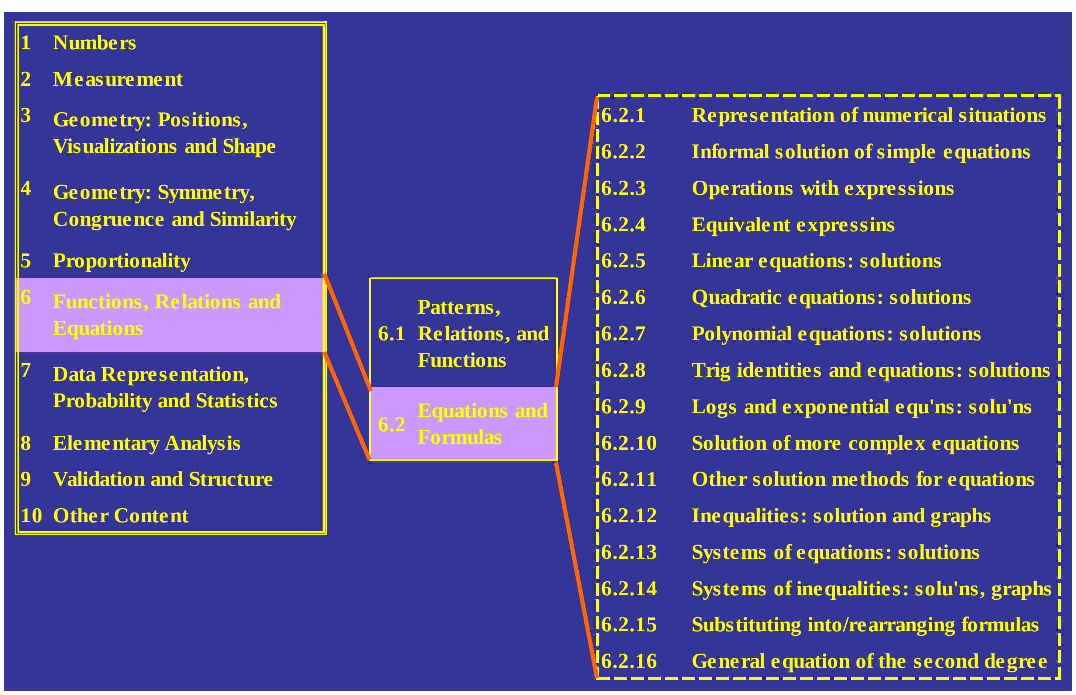
TIMSS Document Analysis

- Designed for Grades K - 12 Materials
  - Curriculum Guides
  - Textbooks
  - Tests
- TIMSS Curriculum Framework
  - Mathematics
  - Sciences
  - Curriculum Standards and Guides

TIMSS Framework

- Examines content and performance expectation
- Content examined:
  - ***What*** topics are intended?
  - ***When*** are topics intended?
- Performance expectation examined:
  - What student performances are expected?
- Procedure is designed to be ***Low Inference***

TIMSS Mathematics Framework



Curriculum Statistical Indicators

- Cumulative signatures of test items
  - Content Topic and Performance Expectation Signature for each item
- Derived Percent of coverage for each content topic

General Topic Trace Mapping

- Part of Data Collection Procedure for Curriculum Analysis
- Each country, district, state, reported when, how long each topic was included in the K-12 curriculum
  - When topic was introduced
  - When topic was focused
- International Grade Placement Index
  - “Average” or composite curriculum

Two Topic Trace Maps

These data are typical topic trace maps for a sample of countries selected to show representative diversity. The results are typical of those for other topics and countries.

Congruence and Similarity

	Ages												
	6	7	8	9	10	11	12	13	14	15	16	17	18
Argentina									+				
Canada	-	-	-	-	-	-	-	-	-	-	-	-	
Cyprus							+		-	+			
Denmark							+	-	-	+	+	-	
Hungary			-	-	-	-	-	+	-	-	+	-	
Iceland								+	-	-	+	-	
Iran						+	-	-	+	-	-		
Ireland								+	+	+		-	+
Japan					+	+		+	-				
New Zealand							-	-	-	-	-		
Spain						+	+						
Tunisia								+	-	-	-	-	+
USA						-	-	+	-	+			

Equations and Formulas

	Ages													
	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Argentina											+	+	-	
Canada					-	-	-	-	+	+	+	+	+	-
Cyprus		-	-	-	-	-	-	-	+	+	+	+	+	
Denmark			-	-	-	+	-	-	+	+	+	+	+	
Hungary				-	-	+	-	-	+	-	-	+		
Iceland									-	-	+	-	+	
Iran								+	-	-	-	+		
Ireland	-	-	-	-	-	-	-	-	+	+	+		+	
Japan				+	+	+	+	+	+	+	+	+	+	
New Zealand	-	-	-	-	-	-	-	-	-	+	+	+	+	
Spain								-	-	-	-	-	-	
Tunisia										-	+	+	+	
USA			-	-	-	-	-	+	+	+	-	+	+	

Examples of the IGP Index

Code	Description	IGP Index
1111	Whole Number: Meaning	1.8
1122	Decimal Fractions	4.6
1141	Binary Arithmetic &/or Other Number Bases	6.6
1143	Complex Numbers & Their Properties	10.7
141	Geometry: Transformations	7.1
151	Proportionality Concepts	6.4
161	Patterns, Relations & Functions	9.0
162	Equations & Formulas	7.0
171	Data Representation & Analysis	7.4
172	Uncertainty & Probability	9.7

The Empirical Challenge is …

A math test is a *NOT* math test

Publisher 1 …

	Grade								
	2	3	4	5	6	7	8	9	10
Numbers									
Whole Number									
Meaning	14	10	12	7	4	5	3	2	1
Operations	65	75	68	74	39	22	29	28	33
Fractions & Decimals									
Common Fractions	1	2	8	15	21	21	21	24	24
Percentages	0	0	0	0	0	8	8	10	12
Integer, Rational & Real Numbers									
Negative Numbers, Integers & Their Properties	0	0	0	0	1	11	10	10	10
Other Numbers & Number Concepts									
Exponents, Roots & Radicals	0	0	0	0	1	1	3	6	11
Measurement									
Units	7	11	24	14	5	5	5	4	5
Functions, Relations, & Equations									
Patterns, Relations & Functions	6	2	0	1	2	1	2	1	2
Data Representation, Probability, & Statistics									
Data Representation & Analysis	11	11	23	23	9	15	12	15	15

Publisher 2 …

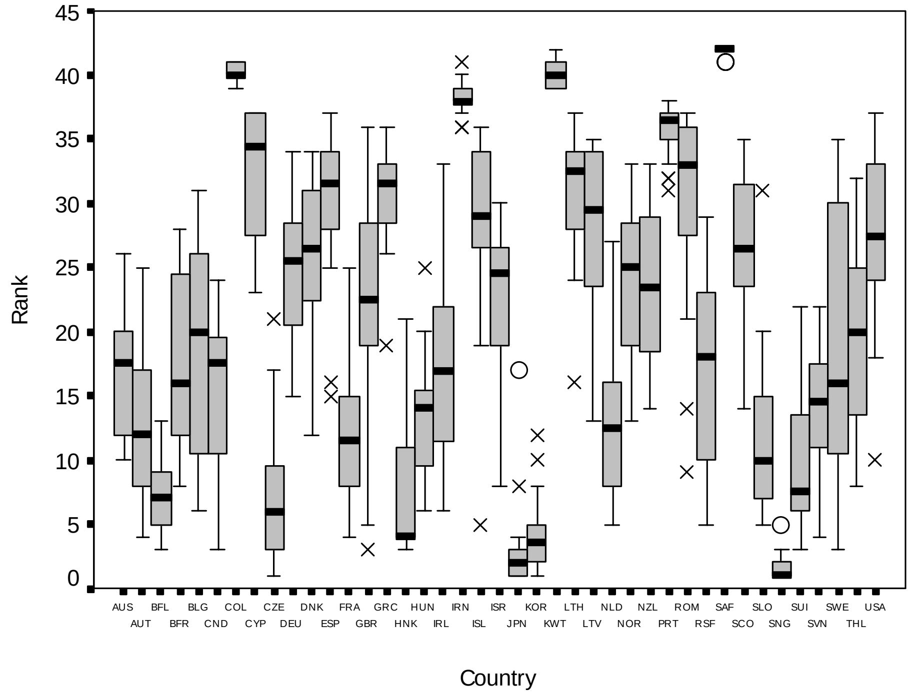
	Grade							
	2	3	4	5	6	7	8	9
Numbers								
Whole Number								
Meaning	16	10	8	4	6	0	1	0
Operations	49	42	42	16	20	11	8	2
Fractions & Decimals								
Common Fractions	4	1	3	16	19	20	17	0
Decimal Fractions	0	4	6	14	10	11	10	2
Measurement								
Units	12	14	10	10	9	11	7	10
Perimeter, Area & Volume	0	1	1	4	4	5	4	18
Geometry: Position, Visualization & Shape								
2-D Geometry: Polygons & Circles	4	4	4	5	4	6	4	18
Functions, Relations, & Equations								
Equations and formulas	9	22	12	29	26	32	19	37
Data Representation, Probability, & Statistics								
Data Representation & Analysis	7	6	7	13	11	15	10	25

4th and 8th Grade Assessments from 6 states …

	4	8	4	8	4	8	4	8	4	8	4	8
Numbers												
Whole Number												
Meaning	1%		26%	8%	9%	3%	5%	1%	4%	3%	8%	
Operations	11%	3%	32%	43%	41%	36%	44%	18%	8%		39%	6%
Fractions & Decimals												
Common Fractions	1%		4%	15%	5%	13%	1%	8%	7%	5%	3%	18%
Decimal Fractions	11%		2%	23%		8%	2%	6%	1%	3%	9%	12%
Relationships of Common & Decimal Fractions			2%	3%		4%		1%			1%	5%
Other Numbers & Number Concepts												
Number Theory	5%		6%	5%		3%	3%	1%	1%		1%	1%
Systematic Counting	7%	1%	13%	5%		1%						
Estimation & Number Sense												
Rounding & Significant Figures	3%		4%	18%			7%	5%			8%	7%
Estimating Computations		1%										4%
Measurement												
Units	19%	1%	9%	40%	10%	7%	13%	7%	25%	38%	9%	8%
Perimeter, Area & Volume	7%	5%	4%	8%		3%	4%	9%	5%	14%	1%	4%
Geometry: Position, Visualization & Shape												
2-D Coordinate Geometry			6%	5%		3%	2%	3%	3%	3%	1%	2%
2-D Geometry: Basics	3%	5%	15%	5%		1%		2%		5%	1%	2%
2-D Geometry: Polygons & Circles	11%	3%	19%	18%	4%	4%	11%	15%	15%	14%	4%	6%
3-D Geometry	2%		2%		2%	2%	13%	3%	5%	14%		1%
Geometry: Symmetry, Congruence & Similarity												
Transformations	3%			5%	1%	1%	3%	3%	3%		3%	1%
Congruence & Similarity	1%	1%			1%	3%	1%	1%	4%		1%	
Constructions w. Straightedge & Compass	4%								1%			
Proportionality												
Problems	0%	11%	6%	18%		4%		8%		3%		8%
Functions, Relations, & Equations												
Patterns, Relations & Functions	8%	22%	17%	10%	28%	22%	3%		12%	8%	4%	2%
Equations and formulas	3%	1%	11%	15%	1%	5%	8%	14%	3%	19%	24%	31%
Data Representation, Probability, & Statistics												
Data Representation & Analysis	34%	47%	26%	30%	9%	8%	12%	12%	19%	14%	9%	12%
Uncertainty & Probability	16%	16%	9%	5%	1%	2%	3%	7%	4%	8%	3%	5%

Are test scores sensitive to curriculum?

Distributions of country ranks across mathematics sub-topics



Results of a Simulation Study of Longitudinal Vertical Scaling …

"… This study demonstrates mathematically that the use of such "construct-shifting" vertical scales in longitudinal, value-added models introduces remarkable distortions in the value-added estimates of the majority of educators. These distortions include (1) identification of effective teachers/schools as ineffective (and vice versa) simply because their students' achievement is outside the developmental range measured well by "appropriate" grade-level tests, and (2) the attribution of prior teacher/school effects to later teachers/schools. Therefore, theories, models, policies, rewards, and sanctions based upon such value-added estimates are likely to be invalid because of distorted conclusions about educator effectiveness in eliciting student growth."

(Martineau, 2004)

Curriculum Effect Missing …

" …find evidence …suggesting a contextual effect could result in systematic error (bias) when they are omitted from the model …"

(Ballou, Sanders and Wright, 2003)

Variation in the mathematics content index (IGP) in schools having multiple tracks and schools having single tracks.

	Single-Track Schools		Multiple – Track Schools	
Source	*IGP Variance*	*IGP Variance (%)*	*IGP Variance*	*IGP Variance (%)*
Track	**0.0859**	**19.2**	**0.3404**	**39.9**
School	**0.0623**	**13.9**	**0.2123**	**24.9**
Class	**0.2994**	**66.9**	**0.3006**	**35.2**

Student, classroom, and school variables employed in the three-level HLM analysis of eighth grade mathematics achievement in schools offering multiple types of eighth grade mathematics.

	Multiple – Track Schools	
Effect of students-level variables	*Effect*	*SE*
Race: White (D)	2.41	-7.10
Race: Black (D)	-26.53***	-7.70
Race: Hispanic (D)	-21.01**	-8.02
Race: Asian (D)	-2.50	-8.92
Socioeconomic Status (SES) (C)	-2.85***	-0.44
Effect of classroom-level variables	*Effect*	*SE*
7th grade achievement	1.00*	-0.49
Mean classroom SES	20.18***	-2.69
Class type: Algebra (D)	62.19***	-6.29
Class type: Pre-Algebra (D)	31.05***	-6.97
Effect of school-level variables	*Effect*	*SE*
Mean school SES	-4.97	-3.00
8th grade enrollment	0.01	-0.02
Minority enrollment (%)	-0.06	-0.11
Location: Urban (D)	4.40	-6.06
Location: Rural (D)	19.73**	-6.92

Note: D denotes dummy variables. C denotes centered variable. SE denotes standard errors.

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.